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Strategic Buybacks of Sovereign Debt

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Abstract

We consider a transaction costs model of sovereign debt buybacks. We show that both secret and publicly known buybacks are profitable for the debtor country. Furthermore, the government of the debtor country would like to spend all of its initial endowment to buy back its debt as soon as possible. When the initial endowment of the government is publicly known, the equilibrium outcome of the secret buyback model is the same as in the public buyback model. However, the equilibrium outcomes are different when the initial endowment is private information of the government. Under reasonable conditions, the secondary market price under publicly observable buybacks is lower than the price under secret buybacks. Therefore the government prefers the former over the latter when the initial endowment is not commonly known.

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I. Introduction

Since the works of Bulow and Rogoff (1988, 1991), it has been generally believed that secondary market buybacks of debt are not profitable for the debtor countries. This insight is based on the observation that the debt is repurchased at its average price rather than at the (lower) marginal price, when creditors are treated on an equal footing in debt renegotiation (equal-sharing). Overall efficiency gains benefit creditors so much that the countries are likely to be worse off, unless additional concessions are obtained.¹

This analysis, however, omits important costs associated with default such as trade sanctions and exclusion from the international financial markets.² Krugman (1988), Froot (1989) and Cline (1995) showed that extremely high costs of default may make buybacks beneficial for the debtor country. Several other authors have also discussed the circumstances under which repurchases of sovereign debt make sense. Cohen and Verdier (1995) argued that secret buybacks of debt allow the debtor country to profitably reduce its indebtedness. Rotemberg (1991) pointed out that the bargaining costs between debtor and creditors are increasing when the level of outstanding debt increases; thus both sides may benefit from a buyback. Similarly, Thomas (1996) derives the profitability of buyback from the assumption that debt can impose considerable costs in addition to the monetary transfers made to creditors when the debtor country defaults.

The objective of this paper is to suggest an alternative model of sovereign debt buybacks. In our model, a government of a sovereign country is indebted to many foreign banks. The debt is scheduled to be fully repaid by a prespecified date. Should the government fail to repay all of its debt by that time, the country faces a punishment, which is an increasing function of the total debt in default. This punishment could take a form of trade sanctions or exclusion from the international financial markets. Between now and the final repayment date, the country's debt is traded on the secondary market. In every period, each creditor is forced to sell its entire loan exposure with a positive probability due to some liquidity or

¹See, for example, Dooley (1988), or Sachs (1988).

²An overall view of the debate is presented by Claessens and Diwan (1989).

regulatory constraints. The government is assumed to be a potential buyer of its foreign debt at the secondary market prices before the final repayment period. The market prices of debt are derived as part of an equilibrium of a dynamic game. Our main goal is to explore the profitability of secret and publicly observable buybacks of sovereign debt.

We show that both secret and publicly known buybacks are profitable for the debtor country. Furthermore, the government of the debtor country would like to spend all of its spare resources called *initial endowment* to buy back its debt as soon as possible. When the initial endowment of the government is publicly known, the equilibrium outcome of the secret buyback model is the same as in the public buyback model. The equilibrium outcomes are different, however, when the initial endowment is private information of the government. Under reasonable conditions, the secondary market price under publicly observable buybacks is lower than the price under secret buybacks. Therefore the government prefers the former over the latter when the initial endowment is not commonly known.

Our analysis is conducted in a transaction costs model. Each time the country's debt is traded the buyer has to incur brokerage fees. This constitutes one difference from the previous literature on debt buybacks. Another difference between our model and most of the existing research is the punishment function for country's default on foreign debts. In contrast to the models introduced by Bulow and Rogoff (1988, 1991) where creditors capture a fixed proportion of country's output in case of default, we assume that the country has to bear the costs which are an increasing function of the total debt in default. This is similar to the sanction function in Rotemberg (1991) and Thomas (1996). The difference in our model is its multiperiod setting which allows us to analyze the dynamics of buybacks. Also, the structure of our model permits comparisons between secret and publicly observable buybacks.

The rest of the paper is organized as follows. In Section II, we describe the basic model and analyze a secret and public buybacks. Section III provides a similar analysis when the initial endowment of the government is its private information. Uncertainty regarding the future disposable income of the government is explored in Section IV. Section V contains

conclusions.

II. A Basic Model

Consider a dynamic model with $N+1$ periods. A government of a sovereign country inherits a stock of foreign debt D_0 in period 0, with final repayment date in period N . Every period any outstanding debt accrues interests at rate r . A secondary market for the debt exists in periods 0 through $N-1$, where the country's debt is traded. The possible buyers of debt on the secondary market are banks and other financial institutes, as well as the debtor country's government. The government has an *initial endowment* M_0 in its account in period 0 which can be used to buy back debt in any period before period N . Also, the money left in the government's account earns interests at the same rate r . The secondary market price of a dollar of debt in period i is denoted by p_i , where $i = 0, 1, 2, \dots, N-1$. Let ρ denote the government's payment per dollar of debt in period N . We assume that no further payments will be made by the government beyond period N .

The government owes debt to many risk-neutral creditors. There are also many other risk-neutral potential investors. Anyone (the investors or the government) who buys the debt has to pay a commission equal to $\eta > 0$ times the total purchase to the broker. In any period, a creditor must sell his entire loan exposure with probability $\beta > 0$ due to some liquidity or regulatory constraints. This probability is the same for all creditors and is independent of a creditor's history. After the government buys back its debt, it will not resell it.

Denote D_i the total amount of debt the government owes at the beginning of period i , M_i the money left in the government's account at that time, and B_i the amount of debt the government buys back in that period.

The government does not make any debt repayments (principle or interests) until the last period (period N), even though the government may buy back some of its debt at market price.³ In period N , the government makes a lump-sum repayment R to the creditors. The

³In our model, as we shall see later, it is always better for the government to buy back debt at a discount than to make a debt repayment.

outstanding debt is then $D_N - R$, and $\rho = R/D_N$, of course, $\rho \leq 1$. The government's objective is to maximize

$$Y + M_N - R - S(D_N - R),$$

where Y is the country's *future disposable income* realized in period N and $S(\cdot)$ is the total loss in revenue due to bad credit ratings, sanctions from the creditors, etc., resulting from the total outstanding debt at the end of period N . It is assumed that $S(0) = 0$, $S'(\cdot) > 0$, and $S''(\cdot) > 0$.

II.1 Secret Government Buybacks

We shall first discuss the case when the government's buyback of debt is secret; the creditors do not know whether the purchase is made by the government or potential investors. Therefore, market prices do not change because of the government's buyback. Assume that the government's scale of buyback is relatively small so that its buyback does not exceed the supply of debt due to liquidity reasons.

Suppose that everyone expects the government's payback per dollar of debt to be ρ in period N . Then buying one dollar of debt in period $N-1$ costs $p_{N-1}(1 + \eta)$. This dollar of debt becomes $1 + r$ dollars the next period, and the total payment on it is given by $(1 + r)\rho$. Taking a creditor's discounting into account, competition between buyers implies:

$$p_{N-1}(1 + \eta) = (1 + r)\rho \frac{1}{1 + r},$$

or

$$p_{N-1} = \rho \frac{1}{1 + \eta}. \quad (1)$$

Similarly, from competition between buyers in period $N-2$, we have

$$p_{N-2}(1 + \eta) = \beta(1 + r)p_{N-1} \frac{1}{1 + r} + (1 - \beta)(1 + r)^2 \rho \frac{1}{(1 + r)^2} = \beta p_{N-1} + (1 - \beta)\rho. \quad (2)$$

In general, considering competition in period i , we obtain

$$p_i(1 + \eta) = \beta(1 + r)p_{i+1} \frac{1}{1 + r} + (1 - \beta)\beta(1 + r)^2 p_{i+2} \frac{1}{(1 + r)^2}$$

$$\begin{aligned}
& +(1-\beta)^2\beta(1+r)^3p_{i+3}\frac{1}{(1+r)^3} + \dots \\
& +(1-\beta)^{N-i-2}\beta(1+r)^{N-i-1}p_{N-1}\frac{1}{(1+r)^{N-i-1}} \\
& +(1-\beta)^{N-i-1}(1+r)^{N-i}\rho\frac{1}{(1+r)^{N-i}} \\
= & \beta p_{i+1} + (1-\beta)\beta p_{i+2} + (1-\beta)^2\beta p_{i+3} + \dots \\
& +(1-\beta)^{N-i-2}\beta p_{N-1} + (1-\beta)^{N-i-1}\rho.
\end{aligned} \tag{3}$$

Therefore, in period $i+1$,

$$\begin{aligned}
p_{i+1}(1+\eta) = & \beta p_{i+2} + (1-\beta)\beta p_{i+3} + (1-\beta)^2\beta p_{i+4} + \dots \\
& +(1-\beta)^{N-i-3}\beta p_{N-1} + (1-\beta)^{N-i-2}p_N.
\end{aligned} \tag{4}$$

Multiplying (4) by $1-\beta$, and subtracting the resulting equation from (3), we obtain

$$p_i(1+\eta) - (1-\beta)p_{i+1}(1+\eta) = \beta p_{i+1}. \tag{5}$$

Rearranging, we have

$$p_i = \frac{\beta + (1-\beta)(1+\eta)}{1+\eta}p_{i+1} = \lambda p_{i+1}, \quad \text{for } i = 0, 1, \dots, N-2, \tag{6}$$

where $\lambda = 1 - \frac{\beta\eta}{1+\eta} < 1$. Since $p_i < p_{i+1}$, it is not in the interest of any creditor to sell its loan exposure at the secondary market price unless forced by liquidity or regulatory constraints.

From (6), we have

$$p_i = \lambda p_{i+1} = \lambda^2 p_{i+2} = \dots = \lambda^{N-i-1} p_{N-1} = \frac{\lambda^{N-i-1}}{1+\eta} \rho. \tag{7}$$

Specifically,

$$p_0 = \frac{\lambda^{N-1}}{1+\eta} \rho. \tag{8}$$

Lemma 1 When all buybacks of debt are secret, they occur only in the initial period, i.e., $B_i = 0$, $i = 1, 2, \dots, N-1$.

Proof We first prove that buying back earlier is better. Buying back \$ 1 of debt in period i costs $(1 + \eta)p_i$. If we put this amount of money in the bank for one period, it becomes $(1 + r)(1 + \eta)p_i$. The amount of debt it can buy back in period $i + 1$ is then $\frac{(1+r)(1+\eta)p_i}{(1+\eta)p_{i+1}}$, which is equal to $\lambda(1 + r) < 1 + r$. Since \$ 1 of debt in period i becomes \$ $(1+r)$ of debt in period $i + 1$, delaying the buyback reduces the amount of debt reduction and is thus worse.

Given that buying back earlier is better, if the government is going to buy back some debt, it should do it in period 0. \square

According to the above lemma, the government only buys back its debt in period 0. The intuition behind Lemma 1 is as follows. Observe that the secondary market price of debt increases over time due to a decreasing chance that the buyer will have to sell before he will be repaid. Thus, the government of a debtor country who finds it profitable to buy back its foreign debt at the secondary market price, prefers to do it as early as possible.

Let B_0 be the amount of debt the government buys back in period 0. Then in period N , the total amount of debt the government accumulates is $(D_0 - B_0)(1 + r)^N$. The government chooses B_0 and final debt repayment R by solving the following problem:

$$\max_{B_0, R} \left\{ Y + [M_0 - p_0(1 + \eta)B_0](1 + r)^N - R - S((D_0 - B_0)(1 + r)^N - R) \right\}$$

$$s.t. \quad B_0 \leq D_0$$

$$p_0(1 + \eta)B_0 \leq M_0$$

$$R \leq (D_0 - B_0)(1 + r)^N$$

$$R \leq Y + [M_0 - p_0(1 + \eta)B_0](1 + r)^N$$

Taking the derivative with respect to R , we have a first-order condition

$$-1 + S'((D_0 - B_0)(1 + r)^N - R) = 0 \tag{9}$$

There are three different cases depending on the slope of the punishment function $S(\cdot)$:

Case 1: There exists $D^* > 0$ such that $S'(D^*) = 1$;

Case 2: $S'(D) > 1$ for all $D > 0$;

Case 3: $S'(D) < 1$ for all $D > 0$.

Let us discuss each case in turn.

Case 1.

In this case, assume that Y is large enough to cover the optimal level of R determined below.⁴ From (9), we have

$$(D_0 - B_0)(1 + r)^N - R = D^*. \quad (10)$$

The marginal punishment for the government at D^* is one for one; an increase of debt by one dollar will increase the sanction by one dollar. Since $S''(D) > 0$, the following holds: $S'(D) > 1$ for $D > D^*$ and $S'(D) < 1$ for $D < D^*$. Therefore, at a higher debt level, a reduction of debt by one dollar reduces the sanction by more than one dollar. Meanwhile, at a lower debt level, an increase of debt by one dollar increases the sanction by less than one dollar. Hence, the optimal level of debt for the government is D^* in period N .

The derivative of the government's objective function with respect to B_0 is

$$-p_0(1 + \eta)(1 + r)^N + S'((D_0 - B_0)(1 + r)^N - R)(1 + r)^N. \quad (11)$$

Given (9), $p_0 = \frac{\lambda^N - 1}{1 + \eta} \rho$, and $\rho \leq 1$, (11) is greater than 0. Therefore, the maximization implies that $p_0(1 + \eta)B_0 = M_0$. That is, the government should spend all its money to buy back its debt in period 0, and

$$B_0 = \frac{M_0}{p_0(1 + \eta)}. \quad (12)$$

To determine ρ and p_0 in this problem, note that, from (10), by substituting (12) and (8), we have

$$R = D_N - D^* = (D_0 - B_0)(1 + r)^N - D^*$$

⁴If Y is not large enough, then the situation is similar to Case 2, where the government intends to repay all of its debt at the end but just does not have enough money to do so. Refer to Case 2 for the analysis.

$$= \left(D_0 - \frac{M_0}{p_0(1+\eta)} \right) (1+r)^N - D^* = \left(D_0 - \frac{M_0}{\lambda^{N-1}\rho} \right) (1+r)^N - D^*. \quad (13)$$

By definition,

$$\rho = \frac{R}{D_N} = \frac{D_N - D^*}{D_N} = \frac{\left(D_0 - \frac{M_0}{\lambda^{N-1}\rho} \right) (1+r)^N - D^*}{\left(D_0 - \frac{M_0}{\lambda^{N-1}\rho} \right) (1+r)^N}. \quad (14)$$

The above equation has the following two solutions with respect to ρ :

$$\rho_{1,2} = \frac{M_0 + (D_0 - \frac{D^*}{(1+r)^N})\lambda^{N-1} \pm \sqrt{[M_0 + (D_0 - \frac{D^*}{(1+r)^N})\lambda^{N-1}]^2 - 4D_0\lambda^{N-1}M_0}}{2D_0\lambda^{N-1}}. \quad (15)$$

Two conditions are imposed to ensure that the analysis is interesting.

Condition 1: $D^* < D_0(1+r)^N$.

This condition ensures that, if the government does not pay back or buy back any debt, the accumulated final debt would exceed the optimal level of debt D^* of the country.

If the inequality is reversed, then the government would not pay back any debt at the end (and thus $R = 0$). Since this implies that the repayment ratio (ρ) is equal to 0, the price for the debt is also equal to zero in any period, and the government can buy back the debt without any cost. Thus, in this case, the government owes nothing in the end.

Condition 2: $M_0 < \left(\sqrt{D_0\lambda^{N-1}} - \sqrt{\frac{D^*\lambda^{N-1}}{(1+r)^N}} \right)^2$.

This condition implies that the expression under the square root sign in (15) is positive. To see this, let $a = M_0$, $b = D_0\lambda^{N-1}$, and $c = \frac{D^*\lambda^{N-1}}{(1+r)^N}$. From Condition 1, $b > c$. From Condition 2, $a < b$. The abovementioned expression under the square root can be written as $(a+b-c)^2 - 4ab$. $(a+b-c)^2 - 4ab > 0$ is equivalent to $a+b-c > 2\sqrt{ab}$, or $(\sqrt{b}-\sqrt{a})^2 > c$, or $\sqrt{b}-\sqrt{a} > \sqrt{c}$; that is, $a < (\sqrt{b}-\sqrt{c})^2$, which is Condition 2.

Roughly speaking, the condition says that the government does not have money to buy back enough debt at the beginning (taking discounted prices into consideration) such that the remaining debt accumulates to exactly D^* at the end. If this condition does not hold,

then the government buys enough debt at the beginning such that there is no repayment at the end $R = 0$, and the prices then drop down to zero.

We assume that Conditions 1 and 2 hold throughout the rest of this paper. Given these two conditions, both solutions to (14) given by (15) are positive and less than 1. The former is easy to see, and to see the latter, we have to check, using the notation following Condition 2, that $\rho_2 = \frac{a+b-c+\sqrt{(a+b-c)^2-4ab}}{2b} < 1$. This inequality can be rewritten as $a + b - c + \sqrt{(a + b - c)^2 - 4ab} < 2b$, or $\sqrt{(a + b - c)^2 - 4ab} < b - a + c$. Given that $a < b$, it is equivalent to $(a + b - c)^2 - 4ab < (b - a + c)^2$. After canceling out terms, we have $-4bc < 0$, which is always true. Thus, $\rho_2 < 1$ is always true under Conditions 1 and 2.

Because $0 < \rho_1 < \rho_2 < 1$, we obtain two equilibria characterized by different levels of the payback ratio. In either equilibrium, the government spends all of its money on a debt buyback in the initial period. The government repurchases more debt in the equilibrium characterized by the lower payback ratio ρ_1 than in the equilibrium characterized by the higher payback ratio ρ_2 . In the former equilibrium, the creditors anticipate the government to repurchase more debt at the beginning. Therefore the payback ratio in the last period is expected to be low. Hence, the price is indeed low, which enables the government to repurchase the anticipated large amount of debt. In the other equilibrium, the creditors expect the government to buy back less debt, so the payback ratio in the last period is expected to be higher, and thus the initial period's price is also higher. With a given amount of money, the government can afford to buy back less debt at this higher price. Both equilibria are self fulfilling and time consistent, even though the government prefers the equilibrium associated with a lower payback ratio ρ_1 .

Note that in each of the above equilibria, the requirement on Y is different. In the equilibrium characterized by the smaller payback ratio ρ_1 , the prices are lower. Since the government can buy back a larger amount of debt, the final period repayment is smaller, and therefore, a lower Y is sufficient. However, in the equilibrium given by the larger payback ratio ρ_2 , the prices are higher. Only a smaller amount of debt can be bought back, and the

repayment is larger. Thus, a larger Y is required to cover the final period repayment. If the size of Y is sufficient in the former equilibrium but not in the latter one, then we again have two equilibria: one is the equilibrium associated with ρ_1 , and the other is characterized later in Case 2.

In the current case, let us assume that in both equilibria Y is large enough to cover the final period repayment R given by (13); that is, $Y \geq \left(D_0 - \frac{M_0}{\lambda^{N-1}\rho_2}\right)(1+r)^N - D^*$. The following theorem summarizes our findings in Case 1:

Theorem 1 Assume that there exists $D^* > 0$ such that $S'(D^*) = 1$. The secret buyback model has two equilibria characterized by different levels of the payback ratio ρ_1 and ρ_2 , given by (15). In the equilibrium associated with ρ_i ($i = 1, 2$), the initial secondary market price $p_0 = \frac{\lambda^{N-1}}{1+\eta}\rho_i$, the amount of debt repurchased $B_0 = \frac{M_0}{p_0(1+\eta)}$, and the final period repayment $R = \left(D_0 - \frac{M_0}{p_0(1+\eta)}\right)(1+r)^N - D^*$.

In each of the above equilibria, the government spends all of the money M_0 to repurchase part of its debt in the initial period, and in addition makes a repayment in the final period, reducing the total outstanding debt to its optimal level D^* at the end of the game. The following example illustrates the equilibria in Case 1.

Example Suppose that $N = 10$, $r = 10\%$, $D_0 = 100$, $M_0 = 10$, $\beta = 0.21$, $\eta = 5\%$, and $S(D) = \frac{1}{20}D^2$. A straightforward algebra gives $D^* = 10$, $\lambda = 0.99$, $\rho_1 = 0.114$, and $\rho_2 = 0.956$. Note that Conditions 1 and 2 are satisfied.

For the lower payback ratio $\rho_1 = 0.114$, the initial price $p_0 = 0.100$. In period 0, the government buys back $B_0 = 95.646$ of its debt, using all of the money $M_0 = 10$. The remaining debt of 4.354 grows to 11.293 in the final period due to compounded interests. At that time, the government repays 1.293 and defaults on the remaining debt of 10 at the end of the game, which leads to a repayment ratio of 11.4%. Note that in this equilibrium, the government buys back a large share of the total outstanding debt, which may not be

available from the creditors in one period, since each creditor sells only with probability .21. Therefore, several periods may be necessary to complete this buyback, which leads to a smaller amount of debt repurchased with the fixed amount of money M_0 due to increasing prices. Hence, the payback ratio as well as the initial secondary market price should be revised upwards.

For the higher payback ratio $\rho_2 = 0.956$, the initial price $p_0 = 0.832$. Using the same amount of money, the government can buy back only $B_0 = 11.445$ of its debt. The remaining debt of 88.555 is compounded to 229.689 in the final period. At that time, the government repays 219.689, which gives a payback ratio of 95.6%.

Case 2.

In this case, the first-order condition (9) cannot be satisfied for any R , i.e.,

$$-1 + S'((D_0 - B_0)(1 + r)^N - R) > 0 \quad \text{for } R > 0.$$

The optimal behavior of the government in the final period is to repay as much as possible of the outstanding debt. The government should repay all of its debt ($R = (D_0 - B_0)(1 + r)^N$) if it has the money, i.e., $Y + [M_0 - p_0(1 + \eta)B_0](1 + r)^N \geq (D_0 - B_0)(1 + r)^N$; otherwise, it should repay $R = Y + [M_0 - p_0(1 + \eta)B_0](1 + r)^N$. From the viewpoint of its creditors, this country has good characteristics.

Let us derive the conditions under which the country can repay all of its debt. In this situation, the payback ratio ρ equals 1. From equation (8), $p_0 = \lambda^{N-1}/(1 + \eta)$, which implies that the derivative of the government's objective function with respect to the amount of the initial debt repurchase B_0 given by (11) is positive. Thus the best strategy for the government in the initial period is to buy back as much debt as possible; hence $B_0 = \frac{M_0}{\lambda^{N-1}}$. Therefore, if

$$Y \geq \left(D_0 - \frac{M_0}{\lambda^{N-1}}\right)(1 + r)^N, \quad (16)$$

the government can afford to repay all of its outstanding debt in the last period.

Should inequality (16) be violated, the government cannot afford to repay all of its outstanding debt in the last period. In this situation the expected payback ratio ρ is smaller than 1, and the derivative given by (11) is still positive. So the best strategy for the government in period 0 is again to buy back as much debt as possible; thus, $B_0 = \frac{M_0}{\rho_0(1+\eta)} = \frac{M_0}{\lambda^{N-1}\rho}$. In the last period, the government has a disposable income Y available for repayments. Since we are considering the case where the country cannot pay back all of its debt, it will use all of its disposable income for debt repayment, i.e., $R = Y$, and

$$\rho = \frac{R}{D_N} = \frac{Y}{(D_0 - B_0)(1+r)^N} = \frac{Y}{\left(D_0 - \frac{M_0}{\lambda^{N-1}\rho}\right)(1+r)^N}.$$

Solving the above equation, we have

$$\rho = \frac{Y}{D_0(1+r)^N} + \frac{M_0}{D_0\lambda^{N-1}}.$$

Since the inequality (16) does not hold, it follows that ρ is less than 1.

The following theorem summarizes our findings in Case 2:

Theorem 2 Assume that $S'(D) > 1$ for all $D > 0$. The secret buyback model has a unique equilibrium characterized by a payback ratio

$$\rho = \begin{cases} 1 & \text{if (16) holds;} \\ \frac{Y}{D_0(1+r)^N} + \frac{M_0}{D_0\lambda^{N-1}} & \text{otherwise.} \end{cases}$$

Case 3.

In this case, the first-order condition (9) cannot be satisfied for any R , i.e.,

$$-1 + S'((D_0 - B_0)(1+r)^N - R) < 0 \quad \text{for } R > 0.$$

The optimal behavior of the government in the final period is not to repay any debt at all, i.e., $R = 0$. Thus the payback ratio ρ equals 0, and the secondary market price of debt also becomes zero; the government can buy back all its outstanding debt at no cost. This

kind of countries do not repay their debts. Of course, no one deals with them in the first place. Note that in our model, the function $S(\cdot)$ is exogeneously determined by the country's characteristics; the government cannot manipulate it to its advantage.

II.2 Public Government Buybacks

In the situation where the government's buyback of debt is public information, the amount of repurchase may affect the current and future prices. The distinction between public and secret buybacks in our model is that, even though in both cases M_0 is known,⁵ the amount of actual buyback is known in the public buyback case but not in the secret buyback case. For example, a larger than expected amount of buyback has no effect on the prices in the secret buyback case. However, when the buyback is public, the investors would infer that the government has a smaller than expected amount of final debt in the last period. Since, everyone knows that the government's optimal outstanding debt is fixed at D^* , it would imply a lower payback ratio; thus, the secondary market price of debt would become lower.

Similarly to the secret buyback case, the government still prefers to make all repurchases in the initial period; the results of Lemma 1 continue to hold.

Lemma 2 When all buybacks of debt are public, they occur only in the initial period, i.e., $B_i = 0$, $i = 1, 2, \dots, N-1$.

Proof We again use backward induction. Suppose that in period $N-1$, the government has money M_{N-1} that it can use to buy back its debt. The price of debt in period $N-1$ is again given by (1). Spending \$1 in period $N-1$ would reduce the current debt by $\frac{1}{p_{N-1}(1+\eta)} = \frac{1}{\rho}$, which effectively reduces the final debt in period N by $\frac{1+r}{\rho}$. If the government saves this \$1, it becomes \$ $1+r$ in period N , and it can be used to reduce the final debt by \$ $1+r$. Since $\rho < 1$, spending the money in period $N-1$ is obviously preferred by the government.

⁵This will not be true in the private information model discussed in Section III.

Therefore, the government would spend all of its M_{N-1} in buying back its debt in period $N-1$.

Now suppose that the government always spends all of M_i in buying back its debt in period i , where $i = n, n+1, \dots, N-1$. Then we shall show that in period $n-1$ the government would prefer to spend all of its M_{n-1} in buying back its debt in equilibrium. Suppose not. Let the government spend $m < M_{n-1}$ in buying back its debt in period $n-1$ in equilibrium. Then the amount of debt bought back in period $n-1$ is $B_{n-1} = \frac{m}{p_{n-1}(1+\eta)}$. According to our assumption, the government will spend all of its money in period n , $M_n = (M_{n-1} - m)(1+r)$, in buying back its debt, and thus $B_n = \frac{(M_{n-1} - m)(1+r)}{p_n(1+\eta)}$. In equilibrium, (6) continues to hold. Thus, $p_{n-1} = \lambda p_n < p_n$. Since the government has no more money to buy back its debts in later periods,

$$\begin{aligned} D_N &= [(D_{n-1} - B_{n-1})(1+r) - B_n](1+r)^{N-n} \\ &= \left[D_{n-1} - \frac{m}{p_{n-1}(1+\eta)} - \frac{(M_{n-1} - m)}{p_n(1+\eta)} \right] (1+r)^{N-n+1} \\ &< \left[D_{n-1} - \frac{M_{n-1}}{p_{n-1}(1+\eta)} \right] (1+r)^{N-n+1}. \end{aligned}$$

The last inequality above made use of the fact that $p_{n-1} < p_n$. Therefore, the government would have lower final debt if it spends all of M_{n-1} in buying back its debt in period $n-1$.

From induction, the government should always spend all of its M_i in buying back its debt in period i , where $i = 0, \dots, n, \dots, N-1$. \square

Lemma 2 states that the government interested in repurchasing its debt should do it in the initial period. This conclusion coincides with the secret buyback case we analyzed earlier. Therefore, the overall maximization problem for the government is the same regardless of the buyback being public or secret. Thus, we have the following theorem:

Theorem 3 The equilibria in the public buyback model are the same as those in the secret buyback model.

III. A Model with A Stochastic Initial Endowment

In the previous section, we assumed that the initial resources of the government M_0 are publicly known. In reality, however, the investors may not know exactly how much money is available to the government for the possible debt repurchase. In this section, we shall incorporate such uncertainty into our model.

Suppose that the government has M^* with probability q and 0 with probability $1 - q$ in the initial period. The actual amount of money is the private information of the government, and the investors know only the above probability distribution. All other aspects of the model are the same as in Section II. We shall focus on the case where D^* is well defined and where the government has enough disposable income to cover the optimal payback expense. Again, this model will be analyzed in two different cases – secret buybacks and public buybacks.

III.1 Secret Government Buybacks

In the case of a secret debt buyback, the timing and the amount of government debt repurchase is not known to the public. Let ρ^* denote the payback ratio in the repayment period when the initial endowment is M^* and $\check{\rho}$ denote the payback ratio when this endowment is 0. Therefore, the expected value of the payback ratio, $E(\rho) = q\rho^* + (1 - q)\check{\rho}$. Similarly to Section II, we can derive that

$$p_{N-1} = \frac{E(\rho)}{1 + \eta}.$$

More generally, we have

$$p_i = \frac{\lambda^{N-i-1}}{1 + \eta} E(\rho),$$

and

$$p_0 = \frac{\lambda^{N-1}}{1 + \eta} E(\rho).$$

The above formulas are similar to those in Section II.1; in both situations, the government buyback is secret. The only difference results from the fact that the initial endowment of the

government is publicly known in the model of Section II.1, but remains private information throughout the game here. Correspondingly, it follows from the above formulas that the government wishing to buy back its debt would do so in the initial period. This property is stated in the following lemma; its proof is parallel to Lemma 1.

Lemma 3 When the initial endowment is private information of the government and all buybacks are secret, any debt repurchases occur only in the initial period, i.e., $B_i = 0$, $i = 1, 2, \dots, N-1$.

Now, we will derive $\check{\rho}$, the payback ratio when the initial endowment of the government $M_0 = 0$, and ρ^* , the payback ratio when the initial endowment $M_0 = M^*$.

If $M_0 = 0$, the government cannot afford to buy back any debt, i.e., $B_0 = 0$. Thus, in period N , the total outstanding debt is $D_N = D_0(1+r)^N$ due to accrued interests. Since the defaulting government's optimal level of debt is D^* , it repays $D_N - D^*$ at the end of the game. Hence, the payback ratio is

$$\check{\rho} = \frac{D_N - D^*}{D_N} = \frac{D_0(1+r)^N - D^*}{D_0(1+r)^N}. \quad (17)$$

If $M_0 = M^*$, the government buys back

$$B_0 = \frac{M^*}{p_0(1+\eta)} = \frac{M^*}{\lambda^{N-1}E(\rho)}$$

in the initial period. Thus, in period N , the total outstanding debt is equal to

$$D_N = (D_0 - B_0)(1+r)^N = \left[D_0 - \frac{M^*}{\lambda^{N-1}E(\rho)} \right] (1+r)^N.$$

Again, since the defaulting government's optimal level of debt is D^* , the equilibrium payback ratio ρ^* must satisfy the following equation:

$$\rho^* = \frac{D_N - D^*}{D_N} = \frac{\left[D_0 - \frac{M^*}{\lambda^{N-1}E(\rho)} \right] (1+r)^N - D^*}{\left[D_0 - \frac{M^*}{\lambda^{N-1}E(\rho)} \right] (1+r)^N}, \quad (18)$$

where $E(\rho) = q\rho^* + (1-q)\check{\rho}$.

Equation (18) has two solutions:

$$\rho_{1,2}^* = \frac{A \pm \sqrt{A^2 - 4D_0\lambda^{N-1}q[M^* - D_0\lambda^{N-1}(1-q)\check{\rho}^2]}}{2D_0\lambda^{N-1}q}. \quad (19)$$

where $A \equiv M^* + (D_0 - \frac{D^*}{(1+r)^N})\lambda^{N-1}q - D_0\lambda^{N-1}(1-q)\check{\rho} = M^* + D_0\lambda^{N-1}(2q-1)\check{\rho}$. Let ρ_1^* be the smaller root and ρ_2^* be the larger root.

Observe that since the payback ratio is an increasing function of D_N , the total outstanding debt in period N , we have $\check{\rho} > \rho^*$. Hence, $\rho^* < E(\rho) < \check{\rho}$, since $E(\rho)$ is a weighted average of $\check{\rho}$ and ρ^* .

Recall that Conditions 1 and 2 introduced in Section II.1 are assumed to hold throughout the paper. These conditions, though, do not guarantee that ρ_1^* and ρ_2^* are both positive. The following lemma shows that the smaller root ρ_1^* is positive unless $M^* \leq D_0\lambda^{N-1}(1-q)\check{\rho}^2$.

Lemma 4 $\rho_1^* > 0$ if and only if $M^* > D_0\lambda^{N-1}(1-q)\check{\rho}^2$.

Proof $\rho_1^* > 0$ if and only if both $A > 0$ and $M^* > D_0\lambda^{N-1}(1-q)\check{\rho}^2$. We shall prove that given Condition 2 for $M_0 = M^*$, $M^* < \left(\sqrt{D_0\lambda^{N-1}} - \sqrt{\frac{D^*\lambda^{N-1}}{(1+r)^N}}\right)^2 = D_0\lambda^{N-1}(1 - \sqrt{1-\check{\rho}})^2$, inequality $A > 0$ is implied by $M^* > D_0\lambda^{N-1}(1-q)\check{\rho}^2$.

Note that the smallest q that can satisfy $M^* \geq D_0\lambda^{N-1}(1-q)\check{\rho}^2$ is given by $q_{min} = 1 - M^*/(D_0\lambda^{N-1}\check{\rho}^2)$. It is sufficient to prove that for q_{min} , inequality $A > 0$ holds.

For q_{min} , we have

$$\begin{aligned} A &= M^* + D_0\lambda^{N-1}(2q_{min} - 1)\check{\rho} \\ &= M^* + D_0\lambda^{N-1} \left[2 \left(1 - M^*/(D_0\lambda^{N-1}\check{\rho}^2) \right) - 1 \right] \check{\rho} \\ &= D_0\lambda^{N-1}\check{\rho} - \frac{1}{\check{\rho}}(2 - \check{\rho})M^*. \end{aligned}$$

From Condition 2 for $M_0 = M^*$, we obtain

$$A > D_0\lambda^{N-1}\check{\rho} - \frac{1}{\check{\rho}}(2 - \check{\rho})D_0\lambda^{N-1}(1 - \sqrt{1-\check{\rho}})^2 = \frac{D_0\lambda^{N-1}}{\check{\rho}} \left[\check{\rho}^2 - (2 - \check{\rho})(1 - \sqrt{1-\check{\rho}})^2 \right].$$

Since $\check{\rho}^2 = (1 - \sqrt{1-\check{\rho}})^2(1 + \sqrt{1-\check{\rho}})^2$, and $(1 + \sqrt{1-\check{\rho}})^2 = 2 - \check{\rho} + 2\sqrt{1-\check{\rho}} > 2 - \check{\rho}$, we

conclude that $A > 0$. This completes the proof. \square

When $M^* < D_0 \lambda^{N-1} (1 - q) \check{\rho}^2$, the only possible equilibrium payback ratio is the larger root ρ_2^* . Straightforward calculations show that $\partial \rho_2^* / \partial M^* < 0$, i.e, the more money the government has, the lower the prices. We can verify that for $M^* = D_0 \lambda^{N-1} (1 - q) \check{\rho}^2$, the root ρ_2^* is nonnegative, and for $M^* = 0$, $\rho_2^* = 1$; thus ρ_2^* is indeed an equilibrium payback ratio.

The following theorem summarizes our results in the model with a stochastic initial endowment of the government.

Theorem 4 Assume the initial endowment of the government M_0 is its private information.

When $M^* < D_0 \lambda^{N-1} (1 - q) \check{\rho}^2$, the secret buyback model has a unique equilibrium characterized by payback ratios $\check{\rho}$ if $M_0 = 0$ and ρ_2^* if $M_0 = M^*$.

When $M^* \geq D_0 \lambda^{N-1} (1 - q) \check{\rho}^2$, the secret buyback model has two equilibria characterized by payback ratios $\check{\rho}$ if $M_0 = 0$ and ρ_1^* or ρ_2^* if $M_0 = M^*$.

III.2 Public Government Buybacks

In the case where the government buybacks are publicly observable, they again take place in period 0, revealing the amount of the initial endowment of the government to the public. This is because once the government announces a buy back of its debt, it signals that its endowment is positive.⁶

Let us derive $\check{\rho}$, the payback ratio when the initial endowment of the government $M_0 = 0$, and $\hat{\rho}$, the payback ratio when the initial endowment $M_0 = M^*$. When the initial endowment of the government is zero, the payback ratio $\check{\rho}$ is again given by (17). The secondary market prices of debt are determined by (1) and (6).

⁶We are analyzing a simplified model here, since the government either has M^* , or no money at all. It would be more difficult for the public to infer how much money the government has if both amounts are positive.

When the initial endowment of the government is M^* the calculation of the equilibria is identical to that of Section II.1. By letting $M_0 = M^*$ in (14), we obtain the equation that determines the equilibrium payback ratio:

$$\hat{\rho} = \frac{\left(D_0 - \frac{M^*}{\lambda^{N-1}\hat{\rho}}\right)(1+r)^N - D^*}{\left(D_0 - \frac{M^*}{\lambda^{N-1}\hat{\rho}}\right)(1+r)^N}. \quad (20)$$

Equation (20) has two solutions in the interval $(0, 1)$:

$$\hat{\rho}_{1,2} = \frac{M^* + (D_0 - \frac{D^*}{(1+r)^N})\lambda^{N-1} \pm \sqrt{[M^* + (D_0 - \frac{D^*}{(1+r)^N})\lambda^{N-1}]^2 - 4D_0\lambda^{N-1}M^*}}{2D_0\lambda^{N-1}}. \quad (21)$$

Let the smaller solution in (21) be $\hat{\rho}_1$ and the larger one $\hat{\rho}_2$. These equilibrium payback ratios determine the secondary market prices of debt. The prices will not affect the government when it has no money. When the government has money, however, a higher price means that less debt can be bought back, and therefore, the government is worse off. The following theorem compares the expected payback ratio $E(\rho) = q\rho_2^* + (1-q)\check{\rho}$ in the secret buyback case with $\hat{\rho}$ in the public buyback case.

Theorem 5 Suppose that $M^* < D_0\lambda^{N-1}(1-q)\check{\rho}^2$. Then ρ_2^* is a decreasing function of q . Furthermore, $\hat{\rho}_2 < \rho_2^* < \check{\rho} < 1$.

Proof Denote $\tau = 1/q$ and $B = D_0\lambda^{N-1}\check{\rho}$. Then from (19), ρ_2^* can be simplified as

$$\begin{aligned} \rho_2^* &= \frac{1}{2D_0\lambda^{N-1}} \left[(M^* - B)\tau + 2B + \sqrt{[(M^* - B)\tau + 2B]^2 - 4D_0\lambda^{N-1}(M^* - B\check{\rho})\tau - 4B^2} \right] \\ &= \frac{1}{2D_0\lambda^{N-1}} \left[-(B - M^*)\tau + 2B + \sqrt{(B - M^*)^2\tau^2 - 4M^*\tau(D_0\lambda^{N-1} - B)} \right]. \end{aligned} \quad (22)$$

Therefore,

$$2D_0\lambda^{N-1}\frac{\partial \rho_2^*}{\partial \tau} = -(B - M^*) + \frac{(B - M^*)^2\tau - 2M^*(D_0\lambda^{N-1} - B)}{\sqrt{(B - M^*)^2\tau^2 - 4M^*\tau(D_0\lambda^{N-1} - B)}}. \quad (23)$$

From our assumption, $M^* < D_0\lambda^{N-1}(1-q)\check{\rho}^2 = B(1-q)\check{\rho} \leq B$, so $B - M^* > 0$. If $C \equiv (B - M^*)^2\tau - 2M^*(D_0\lambda^{N-1} - B) > 0$, then it is straightforward to show that (23) is positive. Hence, ρ_2^* is increasing in τ , which implies that ρ_2^* is decreasing in q .

To prove that C is positive, we only need to note that C is a decreasing function of M^* and that $\tau > 1$. Hence,

$$\begin{aligned} C &> (B - M^*)^2 - 2M^*(D_0\lambda^{N-1} - B) \\ &> [B - B(1 - q)\check{\rho}]^2 - 2B(1 - q)\check{\rho}\left(\frac{B}{\check{\rho}} - B\right) \\ &= B^2[(1 - q)^2(1 - \check{\rho})^2 + q^2] > 0. \end{aligned}$$

Therefore, we complete the proof for ρ_2^* being a decreasing function of q .

By inspecting (19) and (21), we know that they give the same solutions when $q = 1$. Therefore, $\rho_2^* > \hat{\rho}_2$ for $q < 1$. Since $D_0\lambda^{N-1} > B > M^*$,

$$-(B - M^*)\tau + \sqrt{(B - M^*)^2\tau^2 - 4M^*\tau(D_0\lambda^{N-1} - B)} < 0.$$

Therefore, from (22), $\rho_2^* < 2B/(2D_0\lambda^{N-1}) = \check{\rho} < 1$. \square

It is an immediate implication of Theorem 5 that the expected payback ratio when the buyback is secret, $E(\rho)$, is greater than $\hat{\rho}_2$, the payback ratio when the government's initial endowment is positive and the buyback is publicly observable. Therefore the secondary market price under a secret buyback is higher than that price under a publicly observable buyback. When the initial endowment is zero, the secondary market price is irrelevant to the government. When the initial endowment is positive, the government is better off by making the buyback publicly observable rather than keeping it secret.

IV. A Model with A Stochastic Future Income

In general, a country's ability to repay its debt is uncertain at the time the government borrows the money. Consider the following situation. The country's disposable income Y in period N can be Y_H (high) with probability q and Y_L (low) with probability $1 - q$. The government may or may not be able to predict the value of its future disposable income Y in advance. (It turns out that the calculations are the same, since in both cases the government will spend all of M_0 in buying back its debt in period 0.) The investors only

know the probability distribution. Other aspects of the model is the same as those in Section II.

Again, we assume that $S'(D^*) = 1$, so that D^* is the ideal level of total debt the government will owe at the end of the game.⁷ Whether the government's buyback is publicly observable or secret, it is clear that conclusions similar to Theorem 1 remain valid; that is, the government will spend all its initial endowment on a debt buyback in period 0.

Since the government spends all of its initial endowment on a buyback in period 0 independent of the level of future disposable income, no additional information is revealed even if the government has private information about its future income. Therefore, the secondary market prices in various periods depend on the expected level of debt payback ratio.

There are three possibilities regarding the values of future disposable income Y which we have to consider. The first one is that the government has enough income to reduce its total outstanding debt to the level of D^* no matter whether Y is high or low. Thus, the payback ratio is the same for both high and low Y . Therefore, equation (14) continues to hold and the analysis is exactly the same as in Section II.1.

The second possibility is that the income Y_H is sufficient to reduce the total debt to the level of D^* , but the Y_L is not sufficient to do so. Let ρ_H be the payback ratio when and ρ_L be the payback ratio when $Y = Y_L$. Let $E(\rho) = q\rho_H + (1 - q)\rho_L$ be the expected payback ratio. We have

$$\rho_H = \frac{D_N - D^*}{D_N} = \frac{\left(D_0 - \frac{M_0}{\lambda^{N-1}E(\rho)}\right)(1+r)^N - D^*}{\left(D_0 - \frac{M_0}{\lambda^{N-1}E(\rho)}\right)(1+r)^N}, \quad (24)$$

and

$$\rho_L = \frac{Y_L}{D_N} = \frac{Y_L}{\left(D_0 - \frac{M_0}{\lambda^{N-1}E(\rho)}\right)(1+r)^N}. \quad (25)$$

Combining (24) and (25), we obtain

$$E(\rho) = q - \frac{qD^* - (1 - q)Y_L}{\left(D_0 - \frac{M_0}{\lambda^{N-1}E(\rho)}\right)(1+r)^N}.$$

⁷Here we consider the equivalent of Case 1 in Section II.1. We leave the consideration of the equivalents of Cases 2 and 3 to the reader, since they are straightforward.

Solving for $E(\rho)$, we have

$$E(\rho) = \frac{M_0 + (qD_0 - \frac{qD^* - (1-q)Y_L}{(1+r)^N})\lambda^{N-1} \pm \sqrt{[M_0 + (qD_0 - \frac{qD^* - (1-q)Y_L}{(1+r)^N})\lambda^{N-1}]^2 - 4qD_0\lambda^{N-1}M_0}}{2D_0\lambda^{N-1}}.$$

Substituting the above expression into equations (24) and (25), we can determine ρ_H and ρ_L .

The last possibility is that even if the future disposable income of the government is Y_H , it is not sufficient to reduce the total outstanding debt to the level of D^* . In this case, ρ_L is given by (25), and ρ_H becomes

$$\rho_H = \frac{Y_H}{D_N} = \frac{Y_H}{\left(D_0 - \frac{M_0}{\lambda^{N-1}E(\rho)}\right)(1+r)^N}. \quad (26)$$

Combining equations (25) and (26) we obtain the following payback ratios:

$$E(\rho) = \frac{qY_H + (1-q)Y_L}{D_0(1+r)^N} + \frac{M_0}{D_0\lambda^{N-1}},$$

$$\rho_H = \frac{Y_H}{D_0(1+r)^N} + \frac{M_0Y_H}{D_0\lambda^{N-1}(qY_H + (1-q)Y_L)},$$

and

$$\rho_L = \frac{Y_L}{D_0(1+r)^N} + \frac{M_0Y_L}{D_0\lambda^{N-1}(qY_H + (1-q)Y_L)}.$$

As we can see from the analysis in this section, an uncertainty about future disposable income of the government does not affect the qualitative results obtained for the cases when there was no such uncertainty. It is still in the government interest to spend all of its initial endowment on a debt buyback in period 0.

V. Conclusions

In this paper, we demonstrated that the secondary market buybacks of foreign debt are a profitable venture of the debtor country's government in a dynamic transaction costs

model. Comparison between secret and publicly observable buybacks shows no difference in the profitability when the initial endowment is commonly known. However, the publicly observable buyback is more profitable to the debtor country when the initial endowment of the country is private information. This analysis provides a new insight into the issue of profitability of the secondary market buybacks.

In the analysis, we assumed that the debtor country does not have investment opportunities which would bring an above-the-market rate of return. However, our results hold even if the government could earn more than the market rate as long as the return on investment is not too high. There are many examples of secondary market debt buybacks which seem to support our results, but an empirical analysis should be the final judge.

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